MATHEMATICS IN THE UNIVERSITY HIGH SCHOOL

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PREFATORY

If practical teaching is improved through the slow processes of the more or less haphazard classroom experience of teachers working individually, the pace of improvement may evidently be materially quickened and lengthened by an earnest concentration of efforts upon ways and means of effecting improvement. A corps of zealous and admittedly successful practical teachers, believing in advance and determined to effect it, working unitedly and of set purpose upon the problems of secondary mathematics teaching, is a safe guarantee of substantial results, without the abuse of instrumentalities by which results must be accomplished. Where all proposals are subjected to frank and free discussion, criticism, and analysis by persons competent both academically and professionally, under the sole *a priori* limitation that some tangible conclusion must be reached, formulated, and acted upon by all, dogmatic precept meets dogmatic precept to the betterment of both, and the pedagogics of the single point of view melts into and enriches the pedagogics of the larger and more general view. In the University High School the conditions, equipment, and personnel of the department of mathematics unite to make a situation peculiarly competent to the organization and execution of a plan for the very considerable improvement of practical teaching of mathematics in high schools.

REASON FOR SKETCHING THE PROPOSED PLAN

In a school like the School of Education, which is founded upon the principle of pedagogic advance, apology for attempting an improvement in secondary mathematics is unnecessary. It is, however, the very reasonable requirement of friends of the institution that the nature and purpose of proposed improvements be now and then stated. This is all that will be attempted here as to the mathematical undertaking now under way in the University High School.

THREE WEAKNESSES OF THE CURRENT PLAN SPECIFIED

A very little study of the plan of mathematical instruction now generally in vogue in secondary schools reveals, among others, these very serious weaknesses:

By giving a whole year to algebra alone before beginning geometry, as is the custom (and the reverse order of subjects would be only a little better), young pupils are required to take up many very difficult matters in algebra before doing anything with even the easier and very much more vivid concepts of elementary geometry. The damage done to beginners by this procedure would exist in kind, though in much less degree, if the order of algebra and geometry were reversed, by growing the kindred matters of geometry, sometimes inductively, sometimes deductively, and more or less informally, out of arithmetical mensuration, and then passing to the more abstract science of algebra in the second year. A unified body of mathematical truth graduated thoroughly as to intrinsic difficulties is what is needed.

A second weakness of current procedure is that, without the systematic aid, either illustratively or demonstratively, of the more graphic and more visual presentation of the similar, or analogous, mathematical ideas and truths of geometry, it requires that a whole year—and that, too, the most immature year—be given to that particular mathematical subject which in its very nature is the most abstract of all, and is most difficult of all to relate to the life-interests of boys and girls. This weakness would be somewhat mitigated—not removed—by a reversal of the order of algebra and geometry. Correlation of the right sort will largely eliminate it.

A third weakness is that the present scheme isolates, both as to matter and method of treatment, the mathematical subjects from each other, and from the things which to youth seem the realities of life, for so long a time during the early stages of the high-school career as to make it at once discouraging, unscientific, and unpsychological. Geometry being much more readily related to the obvious things of the world and of life, this weakness would be partially relieved by a reversal of the subjects in the curriculum. The only plan promising adequate relief is a broad correlation of the subjects in teaching.

REASON FOR ORGANIZING FIRST-YEAR WORK AROUND ALGEBRA

These are only a few of the ideas that actuated the mathematical faculty to take a step in organizing a body of mathematical matter out of the relevant materials of arithmetic, algebra, and geometry. This, of course, is only a correlation of the mathematical subjects among themselves. Not to depart so far from the present scheme as to render our plan of little interest to other teachers, it was determined to organize our material around an algebraic core. Accordingly, algebra very largely gives trend, unity, and character to the work of the first year. Three months' classroom experience shows the greater wisdom of removing this limitation, although this has not yet been done.

THE FIRST YEAR

The central purpose of the first year's work is to lay a broad and solid foundation of mathematical concepts and elemental truths, and to build solidly upon them in various directions, completing very definitely a considerable body of algebra, and accomplishing meanwhile, on the side, a rounding out, from a higher point of view, of elementary-school mathematics, and doing most of that time-consuming preliminary work necessary to induct beginners into the ideas, method, and spirit of geometry.

Some of the algebraic subjects are given only a first treatment, to be completed later, and many geometrical matters are given a sufficiently full treatment for secondary schools. Unessential details, artificial complexities, and logical over-niceties are omitted, to make room for what is essential and comprehensible to beginners in mathematical reasoning. The thought-values of the work are stressed throughout, the necessary technique being made subsidiary and *auxiliary* to thinking. *Not rules, but reasons*, in the early stages of the work, and later, *when rules become necessary*, *rules with reasons*, are the guiding precepts.

Plan of work on manuscript.—In the preparation of the manuscript these four ideas are agreed upon by its authors:

1. Departure from the well-matured and long-practiced procedure of successful teachers is allowed only when it is substantiated by clear and cogent reasons, and can be seen to promise a distinct gain to the learner. Pure *a priori* argument is not sufficient.

2. Competent and well-supported opinion from every obtainable source, inside or outside of the faculty, as to the scientific and practical merit of proposed plans, is invited and will be fully considered with a view to acting upon it.

3. The body of subject-matter worked out is regarded merely as a *stage of study* of the problem of unifying mathematics in the secondary curriculum.

4. Youths, no less than children, look upon the doing of things as of most worth to them. Accordingly, the principles of mathematics are developed largely through the working of problems and the generalizing of processes. Inductive methods are widely—not exclusively, nor mainly—used.

The first draft of the first-year work is now pretty well completed and is being used in the first-year classes of the University High School. It is also being tried in two or three other good high schools of the country, and with good results.

Outline of topics for first year.—The sectional headings given below will show the topics treated. As stated above, some topics are given only a first treatment, to be carried to greater completeness at a more advanced stage of mathematical maturity.

1. Number generalized.
2. Operations applied to positive numbers (figurate and literal).
3. Operations with numbers represented by lines.
4. Addition of positive and negative numbers.
5. Subtraction of positive and negative numbers.
6. Multiplication of positive and negative numbers.
7. Operations on fractional numbers generalized.
8. Uses of the equation.
9. Uses of inequalities (a rudimentary treatment).
10. The operations of arithmetic abbreviated.
11. The evaluation of algebraic expressions.
12. Drawing to scale.
13. Triangles having the same shape (similar triangles, a first treatment).
14. Equations applied to simple problems on beams.
15. Laws of parallel forces.
16. Laws of turning tendencies (moments).
17. Solutions by the aid of one unknown.
18. The simple equation applied to structural problems.
19. The graph (as a means of picturing functional dependence).

20. The simple equation (formal treatment).
21. Fundamental processes applied to integral expressions.
22. Fundamental processes applied to fractional expressions.
23. Equations involving factorable forms.
24. Fractions involving factorable forms.
25. Factoring.
26. Similarity, ratio, and proportion (a first treatment).
27. Simultaneous linear equations (graphed and solved algebraically).
28. Radicals and irrational forms (a first treatment; square and cube root).
29. Logarithms (a first treatment).

THE SECOND YEAR

In the work of the second year the emphasis of attention shifts from the algebra to the geometry. Many of the algebraic and geometric topics and subjects which start from the same mathematical origin are here pursued algebraically or geometrically, as the work demands, as they diverge naturally into the separate domains of study. In this way much of the subject-matters studied, though correlated in their genesis, is differentiated into the separate fields, and treated scientifically. The pupil has a part in the differentiating process and in the scientific classification.

Building on the conceptual work of the first year, the work of this year is *demonstrative*, and covers plane geometry, with the extensive use of algebraic notation and methods, when they are helpful. The results of geometrical reasoning are stated in precise algebraic language when the content and purport of geometrical conclusions may be made clearer and more precise by so doing. The following, among others, are algebraic subjects which were treated in the *first* year and are given a *fuller* treatment here in connection with the cognate subjects of geometry:

The equation, with congruency propositions of geometry.

Inequalities, with propositions on inequality with geometrical figures.

Ratio and proposition, with similarity.

Quadratics, with propositions depending on the generalized Pythagorean proposition.

Radicals, with computing of sides of regular inscribed or circumscribed figures, etc.

The trigonometry of right triangles is also begun.

THE THIRD YEAR

The third year is given to plane trigonometry with applications, and to advanced algebra, the subjects being carried along together. Plane trigonometry, instead of solid geometry, continues the work of plane geometry.

THE FOURTH YEAR

A course in solid geometry and a course in college algebra are given in the fourth year. These courses are elective and are provided for two purposes: first, as a means of summarizing and differentiating the subject-matter previously studied into the separate subjects; and, second, to prepare for certain technological schools requiring these subjects for admission.

SOME FAULTS IN THE STUDY OF SECONDARY MATHEMATICS AS A LOGICAL SEQUENCE OF PROPOSITIONS

The customary plan of securing the logical value of the secondary mathematical subjects is to require the learner to study a sequence, or chain, of propositions welded together, link by link, day after day. This so soon carries the learner so far away from his base of operations, the axioms, that he quite loses sight of the fundamental character of these primal truths. The average high-school student, no matter how long he has pursued his mathematics, has no realizing sense of the rôle played by the axioms in mathematics, and, as a consequence, he is without standards of judging as to the relative importance of truths as bearing on conclusions being established. Unable to judge of the relevancy of any truth to the argument in question, he flounders in the monotonous mix-up, and loses the major part of that training in judgment without which it is impossible either to marshal arguments and to bring them to bear on a question, or to know when a point has been proved. Thus it comes about that we have in mathematical classes so little of that species of knowledge "that knows that it knows." In this sequential mode, since the strength of the chain is the strength of its weakest link, everything assumes about equal importance, and the clanking of the links of the chain is dinned into the ear of the unwilling pupil in a vain effort to secure a modicum of mechanical exactitude, often mistaken for

logical perfection. Progress on the subject is thus hampered by the circumstance that the pupil's only hope is to keep a firm hold on the chain, and his work may then be justly likened to threading a labyrinth.

The plan now under way in the University High School seeks to a very great extent to deduce the more important mathematical conclusions directly from the axioms. No attempt is made in the initial stages to disentangle and to render precise and consistent a necessary and sufficient body of axioms for algebra and geometry. Nor does it resort to the other alternative, so generally resorted to in the texts, of stating a necessary and sufficient number of axioms and calling upon the pupil to take them on faith. Students are at the outset allowed to use a very rich body of axioms. The body of fundamental notions with which the work proceeds for a time is sufficient, but not necessary. After all is said, axioms and postulates are only *assumed truths*, and no harm is done in allowing many propositions, which seem obvious to the uninitiated, such as "All straight (or right) angles are equal," or "From a point on a line a perpendicular can be drawn," to be taken as axioms—they ought, indeed, to be so taken—until pupils get far enough into the subject both to know and to appreciate what the talk is about when their proof is undertaken. Later these same propositions should be proved. It must be added also that pupils must always be made clearly aware of what is being assumed in an argument.

From what has been said it appears that the pupil assists in the process of disengaging and condensing the elemental truths, and through this assistance he comes to a sense of their fundamental and necessary character. When a step in an argument, or a demonstration, is forgotten, instead of resorting to the broken chain of an author's arbitrary sequence, or, worse still, to a set proof, like a true initiate in the science, the pupil recurs to the axioms and undertakes to draw the forgotten argument from this source. The attitude of mind of the learner is at all times the attitude of the research student, and not that of one being merely instructed. This not infrequently brings the learner upon an unexpected truth and gives him a little of the foretaste of discovery. Only the teacher can appreciate how wholesome an influence this spirit infuses into the class.

SUMMARY

The plan under way in the University High School should then be likened, not to a search for safe anchorage for the end of a chain, so much as to the provision of a rich soil, the careful preparation of it, and the placing of the roots of the seedling so as to assure the soundest and healthiest growth possible, both now and hereafter. It will be seen that this procedure in no way jeopardizes logical interests, and that some distinct gains must accrue to it. Individual initiative is given road and rein, mathematical faith and insight grow into mathematical interest and spirit, while clear and self-reliant thinking promises to mature into the power of independent argumentation and into good judgment.